Supersymmetric contributions to $\bar{B}_s \to \phi\pi^0$ and $\bar{B}_s \to \phi\rho^0$ decays in $SCET$

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(Dated: June 24, 2011)

Using Soft Collinear Effective Theory we analyze the supersymmetric contributions to the branching ratios of $\bar{B}_s \to \phi\pi^0$ and $\bar{B}_s \to \phi\rho^0$ decays. Using mass insertion approximation we show that SUSY contributions mediated by chargino exchange can enhance the branching ratios with respect to the SM prediction. We show that in single mass insertion scenario, Br $(\bar{B}_s \to \phi\pi^0)$ can be enhanced by about 140% with respect to the SM prediction. In two mass insertion scenario we find that Br $(\bar{B}_s \to \phi\pi^0)$ can be enhanced by about 400% with respect to the SM prediction. For Br $(\bar{B}_s \to \phi\rho^0)$ we find that its SM prediction within SCEt is 4.6×10^{-8}. Including SUSY contribution, we show that in single mass insertion scenario Br $(\bar{B}_s \to \phi\rho^0)$ can be enhanced by about 130% with respect to the SM prediction. In two mass insertion scenario we find that Br $(\bar{B}_s \to \phi\pi^0)$ can be enhanced by about 160% with respect to the SM prediction.

PACS numbers: 13.25.Hw,12.60.Jv,11.30.Hv

I.   INTRODUCTION

The decay modes $B \to K\pi$, $\bar{B}_s \to \phi\pi^0$ and $\bar{B}_s \to \phi\rho^0$ are generated at the quark level via $b \to s$ transition. Their amplitudes receive contributions from isospin violating electroweak (EW) penguin amplitudes. However, these contributions are expected to be small in the case of $B \to K\pi$ that receive large contributions from isospin conserving QCD penguins amplitudes which are absent in $\bar{B}_s \to \phi\pi^0$ and $\bar{B}_s \to \phi\rho^0$ decays. Within SM, EW penguin amplitudes are small and hence the predicted branching ratios (Br) of $\bar{B}_s \to \phi\pi^0$ and $\bar{B}_s \to \phi\rho^0$ decays are so small. As a consequence, sizeable enhancement of these branching ratios will be attributed only to isospin-violating new physics which can shed light on the $K\pi$ puzzle[1, 2].

Supersymmetry (SUSY) is one of the best candidates for physics beyond SM. SUSY provides solution to the hierarchy problem. Moreover, SUSY provides new weak CP violating phases which can account for the baryon number asymmetry and other CP violating phenomena in B and K meson decays [3–9]. In addition, the effect of these phases has been studied in the CP asymmetries of τ decays in Refs.([10–12]).

The decay modes $\bar{B}_s \to \phi\pi^0$ has been studied within SM in framework of QCD factorization in Ref.([13]), in PQCD in Ref.([14]) and in Soft Collinear Effective Theory (SCET) in Ref.([15]). On the other hand, the decay modes $\bar{B}_s \to \phi\pi^0$ and $\bar{B}_s \to \phi\rho^0$ has been studied within supersymmetry (SUSY) in framework of QCDF [2, 16]. In this paper we study SUSY contributions to the branching ratios of the $\bar{B}_s \to \phi\pi^0$ and $\bar{B}_s \to \phi\rho^0$ decays in the framework of SCET[17–20].

SCET is an effective field theory describing the dynamics of highly energetic particles moving close to the light-cone interacting with a background field of soft quanta[21]. It provides a systematic and

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rigorous way to deal with the decays of the heavy hadrons that involve different energy scales. Moreover, the power counting in SCET helps to reduce the complexity of the calculations and the factorization formula provided by SCET is perturbative to all powers in α_s expansion.

In SCET, we start by defining a small parameter λ as the ratio of the smallest and the largest energy scales in the given process. Accordingly, we scale all fields and momenta in terms of λ. Then, the QCD lagrangian is matched into the corresponding SCET Lagrangian which is usually written as a series of orders of λ. The smallness of λ allows us to keep terms up to order λ^2 in the SCET Lagrangian which in turn simplifies the calculations.

We can classify two different effective theories: SCET_I and SCET_{II} according to the momenta modes in the process under consideration. SCET_I is applicable in the processes in which the momenta modes are the collinear and the ultra soft as in the inclusive decays of a heavy meson such as $B \to X_s^* \gamma$ at the end point region and $e^- p \to e^- X$ at the threshold region in which there are only collinear and ultra soft momenta modes. SCET_{II} is applicable to the semi-inclusive or exclusive decays of a heavy meson such as $B \to D\pi$, $B \to K\pi$, $B \to \pi\nu e$,....etc in which there are only collinear and soft momenta modes.

This paper is organized as follows. In Sec. II, we briefly review the decay amplitude for $B \to M_1 M_2$ within SCET framework. Accordingly, we analyze the branching ratios of $\bar{B}_s \to \phi\pi^0$ and $\bar{B}_s \to \phi\rho^0$ decays within SM and SUSY in section III. We give our conclusion in Sec. IV.

II. $B \to M_1 M_2$ IN $SCET$

At leading order in α_s expansion, the amplitude of $B \to M_1 M_2$ where M_1 and M_2 are light mesons in SCET can be written as

$$\mathcal{A}^{SCET}_{B \to M_1 M_2} = \mathcal{A}^{LO}_{B \to M_1 M_2} + \mathcal{A}^{\chi}_{B \to M_1 M_2} + \mathcal{A}^{c.c}_{B \to M_1 M_2} \tag{1}$$

Here $\mathcal{A}^{LO}_{B \to M_1 M_2}$ denotes the leading order amplitude in the expansion $1/m_b$, $\mathcal{A}^{\chi}_{B \to M_1 M_2}$ denotes the chirally enhanced penguin amplitude generated by corrections of order $\alpha_s(\mu_h)(\mu_M \Lambda/m_b^2)$ where μ_M is the chiral scale parameter and $\mathcal{A}^{c.c}_{B \to M_1 M_2}$ denotes the long distance charm penguin contributions. For detail discussion about the formalism of SCET we refer to Refs.[15, 17–20, 22].

The decay modes $\bar{B}_s \to \phi\pi^0$ and $\bar{B}_s \to \phi\rho^0$ receive contributions only from $\mathcal{A}^{LO}_{B \to M_1 M_2}$ and so we give a brief review for this amplitude in the following.

At leading power in $(1/m_b)$ expansion, the full QCD effective weak Hamiltonian of the $\Delta_B = 1$ decays is matched into the corresponding weak Hamiltonian in $SCET_I$ by integrating out the hard scale m_b. Then, the $SCET_I$ weak Hamiltonian is matched into the weak Hamiltonian $SCET_{II}$ by integrating out the hard collinear modes with $p^2 \sim \Lambda m_b$ and the amplitude of the $\Delta_B = 1$ decays can be obtained via [23]:

$$\mathcal{A}^{LO}_{B \to M_1 M_2} = -i\langle M_1 M_2 | H^{SCET_{II}}_W | \bar{B}\rangle$$

$$= \frac{G_F m_B^2}{\sqrt{2}}\Big(f_{M_1}[\int_0^1 dudzT_{M_1 J}(u,z)\zeta_J^{BM_2}(z)\phi_{M_1}(u)$$

$$+ \zeta^{BM_2}\int_0^1 duT_{M_1\zeta}(u)\phi_{M_1}(u)] + (M_1 \leftrightarrow M_2)\Big). \tag{2}$$

The parameters $\zeta^{B(M_1,M_2)}$, $\zeta_J^{B(M_1,M_2)}$ are treated as hadronic parameters and can be determined through the χ^2 fit method using the non leptonic decay experimental data of the branching fractions and CP asymmetries. The hard kernels $T_{(M_1,M_2)\zeta}$ and $T_{(M_1,M_2)J}$ are expressed in terms of $c_i^{(f)}$ and $b_i^{(f)}$ which are functions of the Wilson coefficients as follows [22]

$$T_{1\zeta}(u) = \mathcal{C}^{BM_2}_{u_L}\mathcal{C}^{M_1}_{f_L u}c_1^{(f)}(u) + \mathcal{C}^{BM_2}_{f_L}\mathcal{C}^{M_1}_{u_L u}c_2^{(f)}(u)$$

$$+\mathcal{C}^{BM_2}_{f_L}\mathcal{C}^{M_1}_{u_R u}c_3^{(f)}(u) + \mathcal{C}^{BM_2}_{q_L}\mathcal{C}^{M_1}_{f_L q}c_4^{(f)}(u),$$

$$T_{1J}(u,z) = \mathcal{C}^{BM_2}_{u_L}\mathcal{C}^{M_1}_{f_L u}b_1^{(f)}(u,z) + \mathcal{C}^{BM_2}_{f_L}\mathcal{C}^{M_1}_{u_L u}b_2^{(f)}(u,z)$$

$$+\mathcal{C}^{BM_2}_{f_L}\mathcal{C}^{M_1}_{u_R u}b_3^{(f)}(u,z) + \mathcal{C}^{BM_2}_{q_L}\mathcal{C}^{M_1}_{f_L q}b_4^{(f)}(u,z) \tag{3}$$

here f stands for d or s and $\mathcal{C}_i^{BM}$ and $\mathcal{C}_i^M$ are Clebsch-Gordan coefficients that depend on the flavor content of the final states and $c_i^{(f)}$ and $b_i^{(f)}$ are given by [24]

$$c_{1,2}^{(f)} = \lambda_u^{(f)}\Big[C_{1,2} + \frac{1}{N}C_{2,1}\Big] - \lambda_t^{(f)}\frac{3}{2}\Big[\frac{1}{N}C_{9,10} + C_{10,9}\Big] + \Delta c_{1,2}^{(f)},$$

$$c_3^{(f)} = -\frac{3}{2}\lambda_t^{(f)}\Big[C_7 + \frac{1}{N}C_8\Big] + \Delta c_3^{(f)},$$

$$c_4(f) = -\lambda_t^{(f)}\Big[\frac{1}{N}C_3 + C_4 - \frac{1}{2N}C_9 - \frac{1}{2}C_{10}\Big] + \Delta c_4^{(f)}, \tag{4}$$

and

$$b_{1,2}^{(f)} = \lambda_u^{(f)}\Big[C_{1,2} + \frac{1}{N}\Big(1 - \frac{m_b}{\omega_3}\Big)C_{2,1}\Big] - \lambda_t^{(f)}\frac{3}{2}\Big[C_{10,9} + \frac{1}{N}\Big(1 - \frac{m_b}{\omega_3}\Big)C_{9,10}\Big] + \Delta b_{1,2}^{(f)},$$

$$b_3^{(f)} = -\lambda_t^{(f)}\frac{3}{2}\Big[C_7 + \Big(1 - \frac{m_b}{\omega_2}\Big)\frac{1}{N}C_8\Big] + \Delta b_3^{(f)},$$

$$b_4^{(f)} = -\lambda_t^{(f)}\Big[C_4 + \frac{1}{N}\Big(1 - \frac{m_b}{\omega_3}\Big)C_3\Big] + \lambda_t^{(f)}\frac{1}{2}\Big[C_{10} + \frac{1}{N}\Big(1 - \frac{m_b}{\omega_3}\Big)C_9\Big] + \Delta b_4^{(f)}, \tag{5}$$

where $\omega_2 = m_b u$ and $\omega_3 = -m_b\bar{u}$. u and $\bar{u} = 1 - u$ are momentum fractions for the quark and antiquark $\bar{n}$ collinear fields. The $\Delta c_i^{(f)}$ and $\Delta b_i^{(f)}$ denote terms depending on α_s generated by matching from H_W. The $\mathcal{O}(\alpha_s)$ contribution to $\Delta c_i^{(f)}$ has been calculated in Refs.([20, 25, 26]) and later in Ref. ([22]) while the $\mathcal{O}(\alpha_s)$contribution to $\Delta b_i^{(f)}$ has been calculated in Refs.([22, 27, 28]).

III. NUMERICAL ANALYSIS AND RESULTS

In this section, we analyze the branching ratios for $\bar{B}_s \to \phi\pi^0$ and $\bar{B}_s \to \phi\rho^0$ decays. We take $m_b = 4.7\,\text{GeV}$ and the Wilson coefficients C_i at leading logarithm order are are given by [29]:

$$C_{1-10}(m_b) = \{1.110, -0.253, 0.011, -0.026, 0.008, -0.032, 0.09\times10^{-3}, 0.24\times10^{-3},$$

$$-10.3\times10^{-3}, 2.2\times10^{-3}\}. \tag{6}$$

For the other hadronic parameters, we use the same input values given in Ref.([22].

At quark level, the decay modes $\bar{B}_s \to \phi\pi^0$ and $\bar{B}_s \to \phi\rho^0$ are generated via $b \to s$ transition and hence we can decompose their amplitudes $\mathcal{A}$ according to the unitarity of the CKM matrix as

$$\mathcal{A}(\bar{B}_s \to \phi\pi^0, \phi\rho^0) = \lambda_u^s(\mathcal{A}_u^{tree} + \mathcal{A}_u^{EW}) + \lambda_c^s \mathcal{A}_c^{non-cc} \tag{7}$$

Here $\lambda_p^s = V_{pb}V_{ps}^*$ with $p = u, c$ and $\mathcal{A}_u^{tree}, \mathcal{A}_u^{EW}$ refer to the color suppressed tree and Electroweak penguins amplitudes respectively. $\mathcal{A}_c^{non-cc}$ refers to contributions from Electroweak penguins which are proportional to λ_c^s. In the SM, without including QCD corrections, we find that $\mathcal{A}_u^{tree} \gg \mathcal{A}_u^{EW}, \mathcal{A}_c^{non-cc}$ due to the hierarchy of the Wilson coefficients $C_2 \gg C_{3-10}$. Thus, we can write, to a good approximation,

$$\mathcal{A}(\bar{B}_s \to \phi\pi^0, \phi\rho^0) \simeq \lambda_u^s \mathcal{A}_u^{tree} + \lambda_c^s \mathcal{A}_c^{non-cc} \tag{8}$$

which shows as $\mathcal{A}_u^{tree}$ will be suppressed by a factor $|\lambda_u^s| \sim 0.02|\lambda_c^s|$ in comparison to $\mathcal{A}_c^{non-cc}$. Thus we see that Electroweak penguins amplitudes can compete with the tree color suppressed amplitude and one expects that new physics contribution to the penguin amplitude, $\mathcal{A}_c^{non-cc}$, can enhance significantly the decay rates and the branching ratios of these decay modes.

Another remark here is that since these decay modes do not receive contributions from the long distance charm penguin and so we expect very small branching ratios as non-perturbative charming penguin plays crucial rule in the branching ratios using SCET.

A. $\bar{B}_s \to \phi\pi^0$

At leading order in α_s expansion the amplitude of $\bar{B}_s \to \phi\pi^0$ decay including SUSY contributions is given by

$$\begin{aligned}
\mathcal{A}(\bar{B}_s^0 \to \phi\pi^0) \times 10^6 \simeq & (-3.6C_{10} + 1.4\tilde{C}_{10} + 8.3C_7 - 8.3\tilde{C}_7 + 1.9C_8 - 1.9\tilde{C}_8 - 8.3C_9 + 6.6\tilde{C}_9)\lambda_{ts} \\
& + (2.4C_1 - 0.9\tilde{C}_1 + 5.6C_2 - 4.4\tilde{C}_2)\lambda_{us}
\end{aligned} \tag{9}$$

Where C_i and $\tilde{C}_i$ are the Wilson coefficients which can be expressed as

$$C_i = C_i^{SM} + C_i^{SUSY}, \qquad\qquad \tilde{C}_i = \tilde{C}_i^{SUSY} \tag{10}$$

$\tilde{C}_i$ are generated from the weak effective Hamiltonian by flipping the chirality left to right and so in the SM we have $\tilde{C}_i^{SM} = 0$. The dominant SUSY contribution to the Wilson coefficient come from diagrams with exchanging gluino and chargino and so we can write

$$C_i^{SUSY} = C_i^{\tilde{g}} + C_i^{\tilde{\chi}}, \tag{11}$$

where $C_i^{\tilde{g}}$ represents the gluino contribution and $C_i^{\tilde{\chi}}$ represents the chargino contribution.

In SUSY, Flavor Changing Neutral Current(FCNC) and CP quantities are sensitive to particular entries in the mass matrices of the scalar fermions. Thus it is useful to adopt a model independent-parametrization, the so-called Mass Insertion Approximation (MIA) where all couplings of fermions
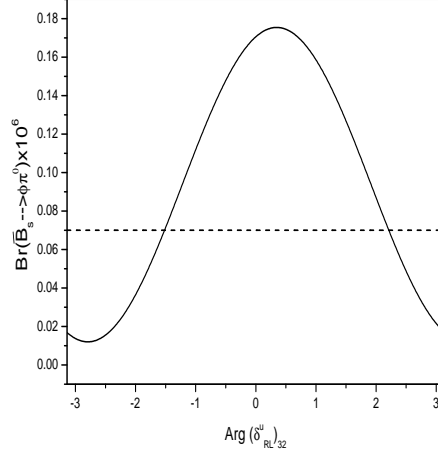
FIG. 1: Br $(\bar{B}_s \to \phi\pi^0) \times 10^6$ versus the phase of the mass insertion $(\delta^u_{RL})_{32}$. The horizontal line in the diagram represents the SM prediction.

and sfermions to neutral gauginos are flavour diagonal [30]. The complete expressions for the gluino and chargino contributions to the Wilson coefficients in MIA can be found in Refs. [3, 31–33]. After substituting with Wilson coefficient and keeping only dominant terms, we get

$$\mathcal{A}(\bar{B}_s \to \phi\pi^0) \times 10^9 \simeq -2.781 - 1.005i - 0.006(\delta^d_{LL})_{23} + 0.005(\delta^d_{RR})_{23} + 0.014(\delta^u_{LL})_{32} + 0.334(\delta^u_{LR})_{31}$$
$$+ \; 1.520(\delta^u_{LR})_{32} - 1.730(\delta^u_{RL})_{32} - 0.381(\delta^u_{RL})_{31} - 0.015(\delta^u_{RR})_{32} \tag{12}$$

Setting all mass insertions to zero which corresponds to SM case, we find $Br(\bar{B}_s \to \phi\pi^0) = 7 \times 10^{-8}$ which is in agreement with prediction in Ref.([15]).

SUSY contribution to $Br(\bar{B}_s \to \phi\pi^0)$ requires non vanishing mass insertions which in general are complex and thus they have new sources for weak phases that can lead to unwanted effects in CP violation phenomena. Therefore, these mass insertions are subjected to constrains from vacuum stability argument [34], experimental measurements concerning FCNC and CP violating phenomena [3]. Recent studies about other possible constraints can be found in Refs.([35–37]).

As can be seen from Eq.(12) the dominant SUSY contributions are those proportional to the mass insertions $(\delta^u_{RL})_{32}$, $(\delta^u_{LR})_{32}$ which come from chargino contributions to Wilson coefficients. Thus we can approximately write

$$\mathcal{A}(\bar{B}_s \to \phi\pi^0) \times 10^9 \simeq -2.781 - 1.005i + 1.520(\delta^u_{LR})_{32} - 1.730(\delta^u_{RL})_{32} \tag{13}$$

The strongest constraints on the mass insertions $(\delta^u_{RL})_{32}$ and $(\delta^u_{LR})_{32}$ are imposed by vacuum stability argument [34, 35]. Applying vacuum stability argument, we find that $|\delta^u_{RL})_{32}| = |\delta^u_{LR})_{32}| \simeq \mathcal{O}(1)$ for squark masses $\leq 700 \; GeV$ and hence to maximize SUSY contributions we can parameterize the mass insertions as $(\delta^u_{RL})_{32} = e^{i\delta_R}$ and $(\delta^u_{LR})_{32} = e^{i\delta_L}$ where δ_R and δ_L are the phases of the mass insertions $(\delta^u_{RL})_{32}$ and $(\delta^u_{LR})_{32}$ respectively.
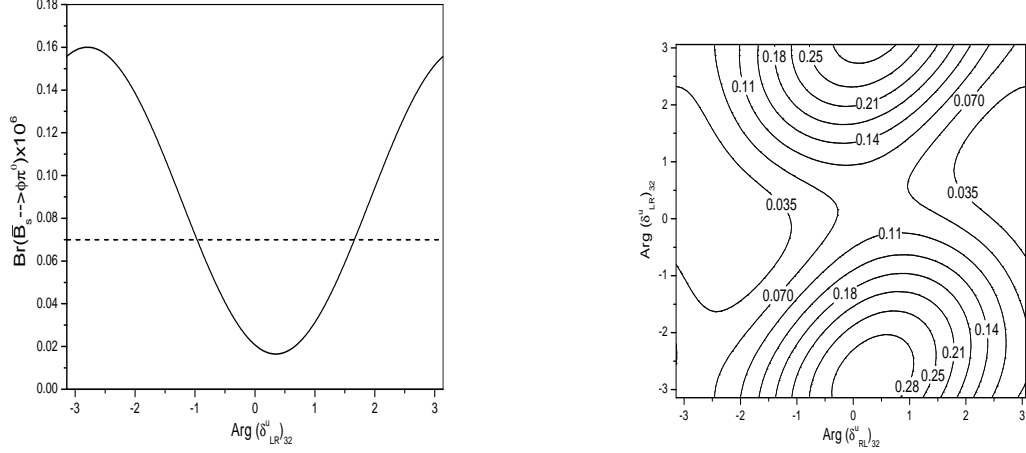
FIG. 2: Left diagram corresponds to Br $(\bar{B}_s \rightarrow \phi\pi^0) \times 10^6$ versus the phase of the $(\delta^u_{LR})_{32}$ where we assume $|(\delta^u_{LR})_{32}| \simeq \mathcal{O}(1)$ and the horizontal line represents the SM prediction. Right diagram corresponds to contours of Br $(\bar{B}_s \rightarrow \phi\pi^0) \times 10^6$ versus the phases of the $(\delta^u_{LR})_{32}$ and $(\delta^u_{RL})_{32}$ for two mass insertions scenario.

In our analysis we consider two scenarios, the first one with a single mass insertion where we keep only one mass insertion per time and take the other mass insertion to be zero and in the second scenario we consider non vanishing two mass insertions.

We start by considering the first scenario and set $(\delta^u_{LR})_{32} = 0$. Thus, in this case, the amplitude in Eq.(13) becomes

$$\mathcal{A}(\bar{B}^0_s \rightarrow \phi\pi^0) \times 10^9 \simeq -2.781 - 1.005i - 1.730(\delta^u_{RL})_{32} \tag{14}$$

The phase of the mass insertion $(\delta^u_{RL})_{32}$, δ_R, can be varied from $-\pi \rightarrow \pi$. Clearly from Eq.(14), last term will have opposite sign to the other terms for the values $\delta_R = \pm\pi$ which leads to a destructive contribution to the amplitude and thus we find Br $(\bar{B}_s \rightarrow \phi\pi^0) \approx 1.6 \times 10^{-8}$ which is less than the SM prediction. For a value $\delta_R = 0$ last term in Eq.(14) will have same sign as other terms. As a consequence, the amplitude will be enhanced and we find that Br $(\bar{B}_s \rightarrow \phi\pi^0) \approx 1.7 \times 10^{-7}$ which is more than 100 % enhancement of the SM prediction. Variation of the Br $(\bar{B}_s \rightarrow \phi\pi^0)$ with δ_R is plotted in Fig.(1) where the horizontal line represents the SM prediction. As can be seen from Fig.(1) that, Br $(\bar{B}_s \rightarrow \phi\pi^0)$ is enhanced by SUSY contribution for δ_R lies in the range $]-1.5, 2.2[$ where the enhancement can reach about 140 % with respect to the SM prediction.

Turning now to the case $(\delta^u_{LR})_{32} \neq 0$ and set the other mass insertion to zeros. In this case the amplitude becomes

$$\mathcal{A}(\bar{B}^0_s \rightarrow \phi\pi^0) \times 10^9 \simeq -2.781 - 1.005i + 1.520(\delta^u_{LR})_{32} \tag{15}$$

As can be seen from Eq.(15), last term will have a sign similar to the other terms for the values $\delta_L = \pm\pi$ which leads to a constructive contribution to the amplitude and thus we find Br $(\bar{B}_s \rightarrow \phi\pi^0) \approx 1.55 \times 10^{-7}$ which is is more than 100 % enhancement of the SM prediction. For a value $\delta_L = 0$, last term in Eq.(15)
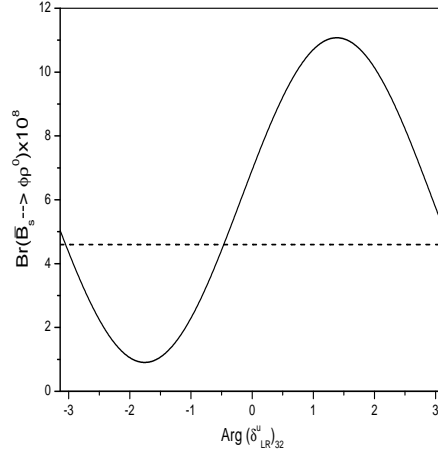
FIG. 3: Br $(\bar{B}_s \to \phi\rho^0) \times 10^8$ versus the phase of the $(\delta^u_{LR})_{32}$. The horizontal line in the diagrams represents the SM prediction.

will have opposite sign as the other terms. As a consequence, the amplitude will be smaller than the SM case and we find that Br $(\bar{B}_s \to \phi\pi^0) \approx 2.1 \times 10^{-8}$ which is less than the SM prediction. Variation of the Br $(\bar{B}_s \to \phi\pi^0)$ with δ_L is plotted in Fig.(2) left where, as before, the horizontal line represents the SM prediction. As can be seen from Fig.(2) left that, Br $(\bar{B}_s \to \phi\pi^0)$ is enhanced by SUSY contribution for all values of δ_L except in the range $[-1, 1.8]$.

We consider now the second scenario by considering two mass insertions per time. As can be seen from Eq.(13) for $\delta_L = \pm\pi$ and $\delta_R = 0$ last two terms will have same sign as the other terms and this will enhance the amplitude and thus we find that Br $(\bar{B}_s \to \phi\pi^0) \approx 3 \times 10^{-7}$ which is enhancement by about 400 % with respect to the SM prediction. For $\delta_L = 0$ and $\delta_R = \pm\pi$ last two terms will have opposite sign to the other terms and this will decrease the amplitude and thus we get Br $(\bar{B}_s \to \phi\pi^0) \approx 1 \times 10^{-8}$. In Fig(2) right, we plot the contours of Br $(\bar{B}_s \to \phi\pi^0)$ as a function of δ_L and δ_R. As can be seen from that Figure, Br $(\bar{B}_s \to \phi\pi^0)$ is enhanced in the regions of the parameter space where the two mass insertions have constructive contributions and is decreased in the regions of the parameter space where the two mass insertions have destructive contributions.

B. $\bar{B}_s \to \phi\rho^0$

The decay mode $\bar{B}^0_s \to \phi\rho^0$ contains two vector mesons in the final state and thus it is characterized by three helicity amplitudes A_0 (longitudinal) and $A_\pm$. Naive factorization analysis leads to the hierarchy $A_0 : A_- : A_+ = 1 : \frac{\Lambda}{m_b} : (\frac{\Lambda}{m_b})^2$ where $m_b \approx 5 GeV$ is the bottom quark mass and $\Lambda \approx 0.5 GeV$ is the strong interaction scale[38]. The hierarchy shows that the dominant contribution is mainly from the longitudinal polarization component. Thus in our calculation we consider only the longitudinal amplitude and so we get
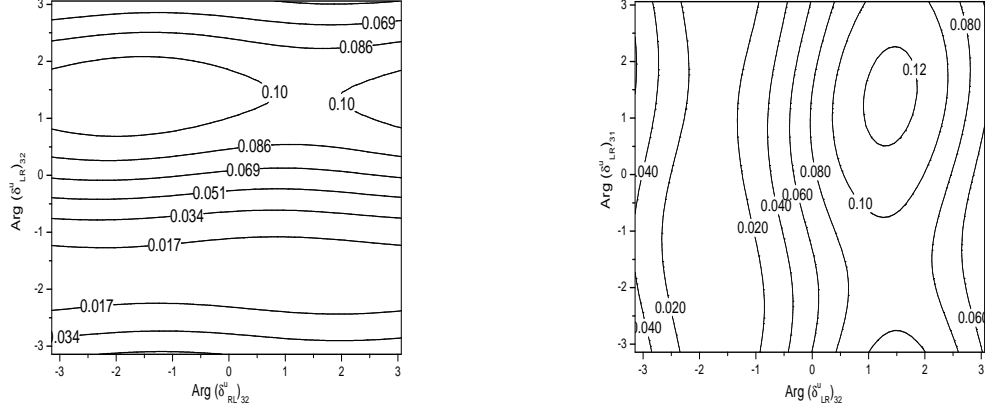
FIG. 4: Contours of Br $(\bar{B}_s \to \phi\rho^0) \times 10^6$ for two mass insertions scenario. Left diagram corresponds to $(\delta^u_{LR})_{32}$ and $(\delta^u_{LR})_{31}$. Right diagram corresponds to $(\delta^u_{LR})_{32}$ and $(\delta^u_{RL})_{32}$

$$\begin{aligned}
\mathcal{A}(\bar{B}_s \to \phi\rho^0) \times 10^6 &\simeq (-4.9C_{10} - 4.9\tilde{C}_{10} - 16.5C_7 + 16.5\tilde{C}_7 - 2.2C_8 + 2.2\tilde{C}_8 - 4.6C_9 - 4.5\tilde{C}_9)\lambda_{ts} \\
&+ (3.3C_1 + 3.2\tilde{C}_1 + 3.1C_2 + 3.0\tilde{C}_2)\lambda_{us}
\end{aligned} \tag{16}$$

We obtain after substituting with Wilson coefficients

$$\begin{aligned}
\mathcal{A}(\bar{B}_s \to \phi\rho^0) \times 10^9 &\simeq -0.445 - 2.353i - 0.003(\delta^d_{LL})_{23} - 0.003(\delta^d_{RR})_{23} - 0.031(\delta^u_{LL})_{32} - 0.293(\delta^u_{LR})_{31} \\
&- 1.330(\delta^u_{LR})_{32} + 0.113(\delta^u_{RL})_{32} + 0.025(\delta^u_{RL})_{31} + 0.030(\delta^u_{RR})_{32}
\end{aligned} \tag{17}$$

Setting all mass insertions to zero which corresponds to SM case, we find $Br(\bar{B}_s \to \phi\rho^0) = 4.6 \times 10^{-8}$.

As can be seen from eq.(17) the dominant SUSY contributions are those proportional to the mass insertions $(\delta^u_{RL})_{32}$, $(\delta^u_{LR})_{32}$ and $(\delta^u_{LR})_{31}$. Thus we can rewrite the amplitude in Eq.(17) as

$$\mathcal{A}(\bar{B}_s \to \phi\rho^0) \times 10^9 \simeq -0.445 - 2.353i - 0.293(\delta^u_{LR})_{31} - 1.330(\delta^u_{LR})_{32} + 0.113(\delta^u_{RL})_{32} \tag{18}$$

We repeat the same analysis as in subsection (III A) by considering two different scenarios one with single mass insertion per time and the other one with two mass insertion per time. We start by considering the first scenario and set all mass insertions to zero except the mass insertion $(\delta^u_{LR})_{32}$. Thus, in this case, the amplitude in Eq.(18) becomes

$$\mathcal{A}(\bar{B}_s \to \phi\rho^0) \times 10^9 \simeq -0.445 - 2.353i - 1.330(\delta^u_{LR})_{32} \tag{19}$$

As can be seen from Eq.(19), last term will have a sign similar to the other terms for the value $\delta_L = 0$ which leads to a constructive contribution to the amplitude and thus we find Br $(\bar{B}_s \to \phi\rho^0) \approx 7 \times 10^{-8}$ which shows that SUSY contribution enhances the SM prediction by about 50%. For the values $\delta_L = \pm\pi$, last term in Eq.(19) will have opposite sign to the other terms. As a consequence, the amplitude will

be smaller than the SM case and we find that Br $(\bar{B}_s \rightarrow \phi\rho^0) \approx 5 \times 10^{-8}$ which is 9% enhancement to the SM prediction. Variation of the Br $(\bar{B}_s \rightarrow \phi\rho^0)$ with δ_L is plotted in Fig.(3) where, as before, the horizontal line represents the SM prediction. As can be seen from the Figure, Br $(\bar{B}_s \rightarrow \phi\rho^0)$ can be enhanced by about 130% with respect to the SM prediction.

Turning now to the case $(\delta_{RL}^u)_{32} \neq 0$ and other mass insertions equal zero. We find in this case, Br $(\bar{B}_s \rightarrow \phi\rho^0)$ can be enhanced only by 5%. We consider now the case $(\delta_{LR}^u)_{31} \neq 0$ and other mass insertions equal zero. It should be noted that single-top production involves the mass insertion $(\delta_{LR}^u)_{31}$ and thus direct top production is the only way to observe or constrain $(\delta_{LR}^u)_{31}$[39]. With unconstrained $(\delta_{LR}^u)_{31}$ we find that the enhancement can reach 20%.

We consider now the second scenario by considering two mass insertions per time and set other mass insertion to zero. We consider first the two mass insertions $(\delta_{LR}^u)_{32}$ and $(\delta_{LR}^u)_{31}$ and set $(\delta_{RL}^u)_{32} = 0$. In this case the amplitude in Eq.(18) is reduced to

$$\mathcal{A}(\bar{B}_s \rightarrow \phi\rho^0) \times 10^9 \simeq -0.445 - 2.353i - 0.293(\delta_{LR}^u)_{31} - 1.330(\delta_{LR}^u)_{32} \tag{20}$$

Clearly from Eq.(20), Br $(\bar{B}_s \rightarrow \phi\rho^0)$ will be enhanced significantly when both $(\delta_{LR}^u)_{32}$ and $(\delta_{LR}^u)_{31}$ have phases ≥ 0 which is shown in Fig.(4) left where we plot the contours of Br $(\bar{B}_s \rightarrow \phi\rho^0)$ versus the phases of the $(\delta_{LR}^u)_{32}$ and $(\delta_{LR}^u)_{31}$. In addition, Br $(\bar{B}_s \rightarrow \phi\rho^0)$ becomes less than SM prediction when both $(\delta_{LR}^u)_{32}$ and $(\delta_{LR}^u)_{31}$ have negative phases. Finally, we see also from the Figure, two mass insertions enhance the branching ratio by about 160% with respect to the SM prediction which is larger than the enhancement obtained in the first scenario with one mass insertion.

Finally, we plot in Fig.(4) right Br $(\bar{B}_s \rightarrow \phi\rho^0)$ as a function of the phases of the two mass insertions $(\delta_{LR}^u)_{32}$ and $(\delta_{RL}^u)_{32}$ where we set $(\delta_{RL}^u)_{31} = 0$. As can be seen from the Figure, Br $(\bar{B}_s \rightarrow \phi\rho^0)$ is also enhanced by about 100% with respect to the SM prediction although the enhancement in this case is smaller than the previous case with the two mass insertions $(\delta_{LR}^u)_{32}$ and $(\delta_{LR}^u)_{31}$.

IV. CONCLUSION

We have analyzed SUSY contributions to the branching ratios of $(\bar{B}_s \rightarrow \phi\pi^0)$ and $(\bar{B}_s \rightarrow \phi\rho^0)$ decays using Soft Collinear Effective Theory. We have adopted in our analysis the mass insertion approximation in two different scenarios. In the first scenario we keep one mass insertion per time and set others to zero while in the second scenario we keep two mass insertions and set others to zero. For squark masses $\leq 700~GeV$, where the constraints on the mass insertions in the up sector are not effective, we have shown that, Br $(\bar{B}_s \rightarrow \phi\pi^0)$ can be enhanced by about 140% with respect to the SM prediction in the first scenario. In two mass insertion scenario we find that Br $(\bar{B}_s \rightarrow \phi\pi^0)$ can be enhanced by about 400% with respect to the SM prediction.

For Br $(\bar{B}_s \rightarrow \phi\rho^0)$ we find that its SM prediction within SCEt is 4.6×10^{-8}. Including SUSY contribution, we have shown that Br $(\bar{B}_s \rightarrow \phi\rho^0)$ can be enhanced by about 130% with respect to the SM prediction in the first scenario. In the second scenario we find that Br $(\bar{B}_s \rightarrow \phi\pi^0)$ can be enhanced by about 160% with respect to the SM prediction.

Acknowledgement

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